Find the point-slope form of the equation of the **normal** line to the curve $y = x^2 \sec x$ at the point where $x = \pi$. **SCORE: _____ / 4 PTS**

$$\frac{dy}{dx} = \boxed{2x\sec x + x^2 \sec x \tan x} \quad 2$$

WHEN $x = \pi$,

$$\left.\frac{dy}{dx}\right|_{x=\pi} = 2\pi(-1) + \pi^2(-1)(0) = \boxed{-2\pi} \quad 1$$

$$y = \pi^2(-1) = -\pi^2$$

$$\boxed{y + \pi^2 = \frac{1}{2\pi}(x - \pi)} \quad 1$$

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If $f(3) = 4$ and $f'(3) = -1$ and $g(x) = x^2 f(x)$, find $g'(3)$. **SCORE: _____ / 3 PTS**

$$g'(x) = \boxed{2x f(x) + x^2 f'(x)} \quad 2$$

$$g'(3) = 2(3)f(3) + 3^2 f'(3)$$

$$= 6(4) + 9(-1)$$

$$= \boxed{15} \quad 1$$

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Prove the derivative of $\csc x$ using the quotient rule. **Show all steps.** **SCORE: _____ / 3 PTS**

$$\frac{d}{dx}\frac{1}{\sin x} = \underset{1}{\boxed{\frac{0 \cdot \sin x - 1 \cdot \cos x}{\sin^2 x}}} = \underset{1}{\boxed{\frac{-\cos x}{\sin^2 x}}} = \boxed{-\frac{1}{\sin x}\,\frac{\cos x}{\sin x}} \quad \tfrac{1}{2}$$

$$= \boxed{-\csc x \cot x} \quad \tfrac{1}{2}$$

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The position of an object at time $t$ is given by $s(t) = 2\sin t - 4\cos t$. **SCORE: _____ / 3 PTS**

Find the acceleration of the object at time $t = \frac{\pi}{3}$.

$$s'(t) = \boxed{2\cos t + 4\sin t} \quad 1$$

$$s''(t) = \boxed{-2\sin t + 4\cos t} \quad 1$$

$$s''\left(\frac{\pi}{3}\right) = -2\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{1}{2}\right) = \boxed{-\sqrt{3} + 2} \quad 1$$

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Prove the quotient rule for derivatives using the definition of the derivative function. **SCORE: _____ / 5 PTS**

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

$$= \lim_{h\to 0}\frac{\frac{f(x+h)-f(x)}{h}\,g(x) - f(x)\frac{g(x+h)-g(x)}{h}}{g(x+h)g(x)}$$

$$= \frac{\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\lim_{h\to 0} g(x) - \lim_{h\to 0} f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}}{\lim_{h\to 0} g(x+h)\lim_{h\to 0} g(x)}$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Find the following derivatives. Simplify all answers appropriately. **SCORE: _____ / 12 PTS**

[a] $\dfrac{d^2}{dx^2}\dfrac{(4x-3)^2}{\sqrt{x}} = \dfrac{d^2}{dx^2}\dfrac{16x^2-24x+9}{x^{\frac{1}{2}}}$

$= \dfrac{d^2}{dx^2}(16x^{\frac{3}{2}} - 24x^{\frac{1}{2}} + 9x^{-\frac{1}{2}})$

$= \dfrac{d}{dx}(\underbrace{24x^{\frac{1}{2}} - 12x^{-\frac{1}{2}} - \tfrac{9}{2}x^{-\frac{3}{2}}}_{2})$

$= \underbrace{12x^{-\frac{1}{2}} + 6x^{-\frac{3}{2}} + \tfrac{27}{4}x^{-\frac{5}{2}}}_{1}$

[b] $\dfrac{d}{db}\dfrac{3b-b^2}{2b+1}$ (Your final answer must be a single fraction)

$= \dfrac{\underbrace{(3-2b)(2b+1)-(3b-b^2)(2)}_{2}}{\underbrace{(2b+1)^2}_{1}}$

$= \dfrac{6b+3-4b^2-2b-6b+2b^2}{(2b+1)^2}$

$= \boxed{\dfrac{-2b^2-2b+3}{(2b+1)^2}}$ 1

[c] $\dfrac{d}{dy}(5e^x + \dfrac{1}{y^e} + 3e^y)$

$= \dfrac{d}{dy}(5e^x + y^{-e} + 3e^y)$

$= \underbrace{-ey^{e-1}}_{1}\underbrace{+3e^y}_{1}$

+1 IF NO OTHER TERMS (IE. NO $5e^x$)

[d] $\dfrac{d}{dx}\dfrac{\cos x - \tan x}{\sin x} = \dfrac{d}{dx}(\cot x - \sec x)$

$= \underbrace{-\csc^2 x}_{1}\underbrace{-\sec x \tan x}_{1}$